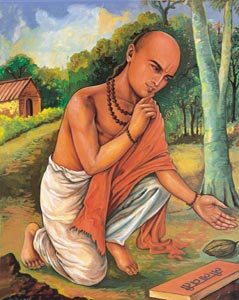
**BHASKARACHARYA – II**



**Bhaskara ii**(1114 – 1185), also known as Bhaskara II and Bhaskara Achārya ("Bhaskara the teacher"), was an Indian mathematician and astronomer. The ii has been appended to his name to distinguish him from the 7th-century astronomer Bhaskara i. He formulated what would go on to become the **foundations of the mathematics** that we use today.

While he came to the wrong conclusions often, Bhaskara’s work served to set the stage for those who would later come along and repurpose his work within their correct frames of reference. Much of his work still continues to be used as-is, without modification, because of how well-thought-out it was. In fact, today, most people still use Bhaskara’s work without even realizing it!

**Early Life**

Labeled as one the **“greatest mathematicians of medieval India”**, the 12th-Century mathematician Bhaskara II wrote many books containing mathematical and astronomical feats which would not be discovered elsewhere for another 500 years.

While very intelligent in his own right, Bhaskara did not develop mathematics himself. He had access to the writing of the Ancient Greek scholars, as well as that of the many Indian mathematicians who had come before him, especially Bhaskara I, a famous Indian mathematician from the 7th Century.

It was Bhaskara I’s influence that inspired Bhaskara II to join Ujjain, India’s “most prestigious mathematical center” at the time. It was at Ujjain where Bhaskara II would formulate the ideas which would go on to become his legacy.

However, he is not well known outside of India. Sure, Bhaskaracharya received high acclaim at home, but he never received much acknowledgment in Europe or the Middle East. Over the years, most of his work went uncredited, or was otherwise given to others.

Bhaskara developed solutions and foundations for what would eventually become calculus, as well as the second proof of the Pythagorean Theorem – yet, no one talks about him outside of his home country.

Still, Bhaskara II’s legacy lives on through other Ujjain mathematicians, and through his son, who established a school dedicated to teaching his work.

## General History of Bhaskara II’s Life

While we may know his work, the man behind it is a mystery. Most of what we now know about Bhaskara comes from his son and the records kept by the Ujjain, where Bhaskara served as head of the astronomical observatory.

From these records, we know that Bhaskara was born in 1114 A.D., near Bijjada Bida (present-day Bijapur), but not much else, especially about his early life. We do know that he was the son of an equally-famous Indian mathematician and astrologer, Mahesvara. Today, many experts agree that it was Mahesvara who taught Bhaskara everything he knew and developed throughout the course of his life.

While at Ujjain, he wrote most of his books. Writing in verse (as was Indian custom at the time), Bhaskara wrote on**several topics of mathematics and astronomy**, such as trigonometry, algebra, and calculus.

## Upon his death in 1185, Bhaskara’s work was picked up by Madhava of Sangamagrama, among other **Indian mathematicians at Kerala School**. These mathematicians would expand upon Bhaskara’s work, to establish the foundations for calculus development in India.

## His Works

Bhaskara developed an understanding of calculus, the number systems, and solving equations, which were not to be achieved anywhere else in the world for several centuries.

Bhaskara is mainly remembered for his 1150 A. D. masterpiece, the Siddhanta Siromani (Crown of Treatises) which he wrote at the age of 36. The treatise comprises 1450 verses which have four segments. Each segment of the book focuses on a separate field of astronomy and mathematics.

They were:

* Lilavati: A treatise on arithmetic, geometry and the solution of indeterminate equations
* Bijaganita: ( A treatise on Algebra),
* Goladhyaya: (Mathematics of Spheres),
* Grahaganita: (Mathematics of the Planets).

He also wrote another treatise named Karaṇā Kautūhala.

### **Lilavati**

Lilavati is composed in verse form so that pupils could memorise the rules without the need to refer to written text. Some of the problems in Leelavati are addressed to a young maiden of that same name. There are several stories around Lilavati being his daughter Lilavati has thirteen chapters which include several methods of computing numbers such as multiplications, squares, and progressions, with examples using kings and elephants, objects which a common man could easily associate with.

Here is one poem from Lilavati:

A fifth part of a swarm of bees came to rest

 on the flower of Kadamba,

 a third on the flower of Silinda

 Three times the difference between these two numbers

 flew over a flower of Krutaja,

 and one bee alone remained in the air,

attracted by the perfume of a jasmine in bloom

 Tell me, beautiful girl, **how many bees were in the swarm**?

**Step-by-step explanation:**

Number of bees- x

A fifth part of a swarm of bees came to rest on the flower of Kadamba - 1/5 x

A third on the flower of Silinda - 1/3 x

Three times the difference between these two numbers flew over a flower of Krutaja –

3 × (1/3−1/5) x

The sum of all bees:

x=1/5x+1/3x+3×(1/3−1/5)x+1

x=8/15x+6/15x+1

1/15x=1

x=15

**Proof:**

3+5+6+1=15

### **Bijaganita**

The Bijaganita is a work in twelve chapters. In Bījagaṇita (“Seed Counting”), he not only used the decimal system but also compiled problems from Brahmagupta and others. Bjiganita is all about algebra, including the first written record of the positive and negative square roots of numbers. He expanded the previous works by Aryabhata and Brahmagupta, Also to improve the Kuttaka methods for solving equations. Kuttak means to crush fine particles or to pulverize. Kuttak is nothing but the modern indeterminate equation of first order. There are many kinds of Kuttaks. For example- In the equation**, a x + b = c y**, a and b are known positive integers, and the values of x and y are to be found in integers. As a particular example, he considered **100x + 90 = 63y**

 Bhaskaracharya gives the solution of this example as, x=18,81,144,207... and y=30,130,230,330... It is not easy to find solutions to these equations. He filled many of the gaps in Brahmagupta’s works.

 Bhaskara derived a cyclic, chakravala method for solving indeterminate quadratic equations of the form **a x2 + b x + c = y.** Bhaskara’s method for finding the solutions of the problem **N x2 + 1 = y2** (the so-called “Pell’s equation”) is of considerable importance.

The book also detailed Bhaskara’s work on the Number Zero, leading to one of his few failures. He concluded that dividing by zero would produce an infinity. This is considered a flawed solution and it would take European mathematicians to eventually realise that dividing by zero was impossible.

Some of the other topics in the book include quadratic and simple equations, along with methods for determining surds.

Touches of mythological allegories enhance Bhaskasa ii’s Bījagaṇita. While discussing properties of the mathematical infinity, Bhaskaracharya draws a parallel with Lord Vishnu who is referred to as Ananta (endless, boundless, eternal, infinite) and Acyuta (firm, solid, imperishable, permanent): During pralay (Cosmic Dissolution), beings merge in the Lord and during sṛiṣhti (Creation), beings emerge out of Him; but the Lord Himself — the Ananta, the Acyuta — remains unaffected. Likewise, nothing happens to the number infinity when any (other) number enters (i.e., is added to) or leaves (i.e., is subtracted from) the infinity. It remains unchanged.

### **Grahaganita**

The third book or the Grahaganita deals with mathematical astronomy**.** The concepts are derived from the earlier works Aryabhata. Bhaskara describes the heliocentric view of the solar systemand the elliptical orbits of planets, based on Brahmagupta’s law of gravity**.**

Throughout the twelve chapters, Bhaskara discusses topics related to mean and true longitudes and latitudes of the planets, as well as the nature of lunar and solar eclipses**.** He also examines planetary conjunctions, the orbits of the sun and moon, as well as issues arising from diurnal rotations.

He also wrote estimates for values such as the**length of the year**, which was so accurate that we were only of their actual value by a minute!

### **Goladhyaya**

Bhaskara’s final, thirteen-chapter publication is all about **spheres and similar shapes**, such as **armillary spheres, rings, and hoops**. Most of the book deals with spherical trigonometry, in which Bhaskara found the sine of many angles, from 18 to 36 degrees. The book even includes a sine table, along with the many relationships between trigonometric functions.

Other topics in the Goladhyaya include:

* Cosmography
* Geography and the seasons
* Planetary mean motion
* Ellipses and lunar crescents

Bhaskara also discussed an epicycle model for planetary orbits. This model proposed that some planets, such as **the sun and moon**, move in small circles as they move in their orbits. While the model does not line up with reality, Bhaskara notes how difficult it is to calculate astronomical phenomena using the astronomical instruments that were available at the time.

## The Bhaskara Equation

Throughout the entire Siddhanta Siromani series, Bhaskara reached many **mathematical achievements**that rivaled the works of both Europe and China.

However, none of these would come close to the equation that now bears his name – despite most people having never heard of it, even though they know what it is and how to use it.

This is because most of the world simply refers to **Bhaskara’s discovery** as the quadratic formula for the solutions of quadratic equations:

X = - b/a ± sqrt(b2-4ac)/a

Originally used just for real roots, others would later expand this solution to include complex number solutions as well, making the quadratic formula one of Bhaskara’s most important discoveries.

Along with the quadratic formula, Bhaskara also created other insights into quadratic equations.

For instance, he also gave us the formula for finding the square root of a sum of a number and a square root:

Sqrt(a ± sqrt(b)) = sqrt(½(a + sqrt(a2 – b)) ± sqrt(½(a – sqrt(a2 – b))

## Other Achievements in Mathematics and Astronomy

While the quadratic formula was big, it was not Bhaskara’s only achievement that he wrote about in his treatises.

### **Spherical Trigonometry**

Bhaskara developed many of the**trigonometric identities and formulas** used throughout mathematics, navigation, geodesy, and astronomy.

For instance, the Goladhyaya is the first publication to report the now-famous formulas:

* sin(a + b) = sin(a) cos(b) + cos(a) sin(b)
* sin(a – b). sin(a) cos(b) – cos(a) sin(b)

### **Rolle’s Theorem**

Aside from that, Bhaskara also made advancements in other areas of math and astronomy, including (but not limited to) his work on what would become **Rolle’s Theorem.**

In particular, Bhaskara noted that the distance between a planet’s real and predicated locations become zero when the planet is at its farthest, or nearest, points to Earth. This solution to what was the center equation hinted towards the general mean value theorem which would replace it.

### Foundations of Calculus

Bhaskara’s other **major contributions** are the foundations of what would later become calculus. In particular, he also dabbled in differential calculus, integral calculus, and mathematical analysis – even though he never fully recognized their usefulness. He just developed the principles to solve problems in astronomy.

While Isaac Newton and [Gottfried Leibniz](https://www.storyofmathematics.com/17th_leibniz.html) would later serve to bring everything together, Bhaskara was the one who formulated the foundations long before they were born. He even coined the concepts of the derivative and differential coefficients.

## Bhaskara II’s Legacy

While the man himself is not well-known outside of India, **Bhaskara’s work traveled the globe**. Other [Indian mathematicians](https://www.storyofmathematics.com/indian.html) picked up his work immediately upon his death, with the establishing of his school. Outside of India, his work got compiled into the works of **Islamic scholars,** only to be reassigned back to him recently.

Still, Bhaskara’s influence on mathematics cannot be denied or understated.

In fact, there is strong evidence that Islamic mathematicians knew about Bhaskara and his ideas as he published them. Many reported Islamic mathematic discoveries to show direct influence, starting from the end of the 12th Century, ultimately culminating in a famous 1587 Persian translation of the Lilavati.

In Europe, Bhaskara’s influence came through those **Islamic channels**. While no one in Europe would know his name until the 20th Century, European scholars were using Bhaskara’s work all throughout the **Renaissance.**

Some scholars do debate whether Bhaskara II was influenced by Diophantine, although these claims were later disputed, as similar Indian developments on Diophantine equations going as far back as the Sulba Sutras were written from 800 to 500 BCE.

Nevertheless, Bhaskara II’s work will continue to inspire other Indians to take up mathematics for centuries to come.